1.

$$f(x) = \frac{50x^2 + 38x + 9}{(5x+2)^2(1-2x)} \qquad x \neq -\frac{2}{5} \quad x \neq \frac{1}{2}$$

Given that f(x) can be expressed in the form

$$\frac{A}{5x+2} + \frac{B}{(5x+2)^2} + \frac{C}{1-2x}$$

where A, B and C are constants

(a) (i) find the value of B and the value of C

(ii) show that $A = 0$

(4)

(b) (i) Use binomial expansions to show that, in ascending powers of x

$$f(x) = p + qx + rx^2 + \ldots$$

where p, q and r are simplified fractions to be found.

(ii) Find the range of values of x for which this expansion is valid.

(7)

a) $f(x) = \dfrac{50x^2 + 38x + 9}{(5x+2)^2(1-2x)}$

i) $\dfrac{50x^2+38x+9}{(5x+2)^2(1-2x)} = \dfrac{A}{5x+2} + \dfrac{B}{(5x+2)^2} + \dfrac{C}{1-2x}$

$50x^2 + 38x + 9 = A(5x+2)(1-2x) + B(1-2x) + C(5x+2)^2$

multiplied by $(5x+2)^2(1-2x)$ 1

sub in $x = \frac{1}{2}$:

$50\left(\frac{1}{2}\right)^2 + 38\left(\frac{1}{2}\right) + 9 = C\left(5\left(\frac{1}{2}\right) + 2\right)^2$

$\frac{81}{2} = \frac{81}{4}C$

$C = 2$

$$50x^2+38x+9 = A(5x+2)(1-2x)+B(1-2x)+C(5x+2)^2$$

sub in $x = -\frac{2}{5}$:

$$50\left(-\frac{2}{5}\right)^2+38\left(-\frac{2}{5}\right)+9 = B\left(1-2\left(-\frac{2}{5}\right)\right)$$

$$\frac{9}{5} = \frac{9}{5}B$$

$$B = 1 \quad ①$$

ii) sub in $x=0$: $9 = 2A + B + 4C$ ①

$$9 = 2A + 1 + 8$$

$$2A = 0$$

$$A = 0 \quad ①$$

$$f(x) = \frac{1}{(5x+2)^2} + \frac{2}{1-2x}$$

b i) $f(x) = (5x+2)^{-2} + 2(1-2x)^{-1}$

$$(5x+2)^{-2} = \left(2\left[\frac{5x}{2}+1\right]\right)^{-2} = 2^{-2}\left(1+\frac{5}{2}x\right)^{-2} \quad ①$$

$$\left(1+\frac{5}{2}x\right)^{-2} = 1 - 2\left(\frac{5}{2}x\right) + \frac{(-2)(-3)}{2!}\left(\frac{5}{2}x\right)^2 + \dots \quad ①$$

$$= 1 - 5x + \frac{75}{4}x^2 + \dots$$

$$\therefore 2^{-2}\left(1+\frac{5}{2}x\right)^{-2} = \frac{1}{4} - \frac{5}{4}x + \frac{75}{16}x^2 + \dots \quad ①$$

$$(1-2x)^{-1} = 1 + 2x + \frac{(-1)(-2)}{2!}(-2x)^2 + \dots \quad (1)$$

$$= 1 + 2x + 4x^2 + \dots$$

$$\therefore\ 2(1-2x)^{-1} = 2 + 4x + 8x^2 + \dots$$

$$f(x) = \frac{1}{(5x+2)^2} + \frac{2}{1-2x}$$

$$= \left[\frac{1}{4} - \frac{5}{4}x + \frac{75}{16}x^2 + \dots\right] + \left[2 + 4x + 8x^2 + \dots\right] \quad (1)$$

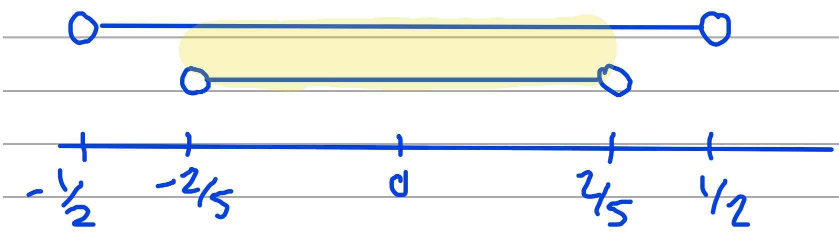
$$= \frac{9}{4} + \frac{11}{4}x + \frac{203}{16}x^2 + \dots \quad (1)$$

b ii) $\left(1+\frac{5}{2}x\right)^{-2}$ is valid for $\left|\frac{5x}{2}\right| < 1$ $\quad \times \frac{2}{5}$

$$|x| < \frac{2}{5}$$

$(1-2x)^{-1}$ is valid for $|-2x| < 1$

$$|x| < \frac{1}{2}$$



want the overlap region, so

$$|x| < \frac{2}{5} \quad (1)$$

2. (a) Express $\dfrac{3}{(2x-1)(x+1)}$ in partial fractions.

(3)

When chemical A and chemical B are mixed, oxygen is produced.

A scientist mixed these two chemicals and measured the total volume of oxygen produced over a period of time.

The total volume of oxygen produced, $V\,\text{m}^3$, t hours after the chemicals were mixed, is modelled by the differential equation

$$\frac{dV}{dt} = \frac{3V}{(2t-1)(t+1)} \qquad V \geqslant 0 \qquad t \geqslant k$$

where k is a constant.

Given that exactly 2 hours after the chemicals were mixed, a total volume of $3\,\text{m}^3$ of oxygen had been produced,

(b) solve the differential equation to show that

$$V = \frac{3(2t-1)}{(t+1)}$$

(5)

The scientist noticed that

- there was a **time delay** between the chemicals being mixed and oxygen being produced
- there was a **limit** to the total volume of oxygen produced

Deduce from the model

(c) (i) the **time delay** giving your answer in minutes,

(ii) the **limit** giving your answer in m^3

(2)

a) $\dfrac{3}{(2x-1)(x+1)} \equiv \dfrac{A}{2x-1} + \dfrac{B}{x+1}$ (1)

$3 \equiv A(x+1) + B(2x-1)$

$x = -1$: $3 = -3B$ $\therefore$ $B = -1$ (1)

$x = \frac{1}{2}$: $3 = \frac{3}{2}A$ $\therefore$ $A = 2$

$\therefore \dfrac{3}{(2x-1)(x+1)} \equiv \dfrac{2}{2x-1} - \dfrac{1}{x+1}$ (1)

b) $\frac{dV}{dt} = \frac{3V}{(2t-1)(t+1)}$, $V \geq 0$, $t \geq k$

$$\int \frac{1}{V}\, dV = \int \frac{3}{(2t-1)(t+1)}\, dt \quad (1)$$

$$= \int \left\{ \frac{2}{2t-1} - \frac{1}{t+1} \right\} dt \quad (1)$$

$$\ln|V| = \ln|2t-1| - \ln|t+1| + c \quad (1)$$

$$\ln|V| = \ln\left|\frac{2t-1}{t+1}\right| + c$$

when $t = 2$ and $V = 3$,

$$\ln 3 = \ln\left(\frac{3}{3}\right) + c$$

$$\ln 3 = \ln 1 + c$$

$$\therefore c = \ln 3 \quad (1)$$

$$\therefore \ln|V| = \ln\left|\frac{2t-1}{t+1}\right| + \ln 3$$

$$\ln|V| = \ln\left|\frac{3(2t-1)}{t+1}\right|$$

$$\therefore V = \frac{3(2t-1)}{t+1} \quad (1)$$

(c) (i) $V = 0$: $\frac{3(2t-1)}{t+1} = 0$

$2t - 1 = 0 \quad \therefore t = \frac{1}{2}$

$\therefore$ 30 minutes (1)

(ii) $V = \frac{6t-3}{t+1}$

$V = \frac{6 - \frac{3}{t}}{1 + \frac{1}{t}}$

when $t \to \infty$, $V = \frac{6-0}{1+0} = 6$

$\therefore$ limit is $6\ m^3$ (1)

3. $\mathrm{f}(x) = \dfrac{3kx - 18}{(x+4)(x-2)}$ where k is a positive constant

(a) Express $\mathrm{f}(x)$ in partial fractions in terms of k.

(3)

(b) Hence find the exact value of k for which

$$\int_{-3}^{1} \mathrm{f}(x)\,\mathrm{d}x = 21$$

(4)

a) $f(x) = \dfrac{3kx-18}{(x+4)(x-2)} = \dfrac{A}{x+4} + \dfrac{B}{x-2}$

$3kx - 18 = A(x-2) + B(x+4)$ ①
$= Ax - 2A + Bx + 4B$
$= (A+B)x + (4B - 2A)$

x: $3k = A + B \Rightarrow B = 3k - A$ ①
constant: $-18 = -2A + 4B$ ②

sub ① into ②: $-18 = -2A + 4(3k - A)$ ①
$-18 = -2A + 12k - 4A$
$6A = 12k + 18$
$A = 2k + 3$

$B = 3k - (2k+3) = k - 3$

$f(x) = \dfrac{2k+3}{x+4} + \dfrac{k-3}{x-2}$ ①

b) $\displaystyle\int_{-3}^{1} f(x)\,dx = 21 \quad \therefore \int_{-3}^{1} \frac{2k+3}{x+4} + \frac{k-3}{x-2}\,dx = 21$

$$\Big[(2k+3)\ln|x+4| + (k-3)\ln|x-2|\Big]_{-3}^{1} = 21$$ ① ①

$$(2k+3)\ln 5 + \cancel{(k-3)\ln 1} - \cancel{(2k+3)\ln 1} - (k-3)\ln 5 = 21$$

$\ln 1 = 0$

$$(2k+3-k+3)\ln 5 = 21$$ ①

$$(k+6)\ln 5 = 21$$

$$k+6 = \frac{21}{\ln 5}$$

$$k = \frac{21}{\ln 5} - 6$$ ①